Math 116b - Homework 7

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`Due:` March 4, 2008 at 1:00 pm.

This Homework is due during lecture by Tuesday March 4 at 1:00 pm. Refer to the grading policy for additional requirements.

Assume a fixed (r.e.) listing of the Turing machines, so $\{e\}$ denotes the e-th machine. Let $\Phi_e(x)$ denote the number of steps the machine with index e uses in computing $\{e\}(x)$ (undefined if $\{e\}(x) \uparrow$).

1. Provide an explicit table for a Turing machine witnessing that $f(x, y) = x \cdot y$ is T-recursive.

2. For any (total) recursive function $f : \mathbb{N} \to \mathbb{N}$ show that there is a recursive function $g : \mathbb{N} \to \{0, 1\}$ such that for any index e for (a machine computing) g,
$$\Phi_e(x) > f(x)$$
for all but finitely many values of x.

3. For any (total) recursive $g : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ show that there is a recursive function $f : \mathbb{N} \to \{0, 1\}$ such that for any index i for (a machine computing) f there is an index j for f such that
$$g(x, \Phi_j(x)) < \Phi_i(x)$$
for all but finitely many values of x.

4. Prove explicitly that a set $A \subseteq \mathbb{N}$ is T-recursive iff both A and $\mathbb{N} \setminus A$ are T-r.e.