Math 117b - Homework 4 (Continued)

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`Due:` February 1, 2007 at 1:00 pm.

Here are some hints for problem 3:

3. Without using that exponentiation is Diophantine, show that if

$$\{(a,b)\in\mathbb{N}^2 : \exists n\,(a=b^n)\}$$

is Diophantine *then* so is exponentiation:

$$\{(a,b,c)\in\mathbb{N}^3 : a=b^c\}.$$

`Hint:` **(J. Robinson)** Let $Pow(x,y)$ iff x is a power of y. So our assumption is that Pow is Diophantine.

Start by showing that if $u=2^z$ then

$$u+z=\left[\!\left[\frac{u(u+1)^z}{u^z}\right]\!\right].$$

(You may find it useful to remember that $u>\binom{z}{k}$ if $z\geq k\geq 2$.)

Let $Same(a,b,c,d)$ be the relation that holds iff there is some n such that $a=b^n$ and $c=d^n$. Assume that $Same$ is Diophantine. Use the equality above to show that the ternary relation $x=y^z$ is Diophantine (for $z>0$, $y>1$).

To show that $Same$ is Diophantine if Pow is, notice that if $b,d>1$, then b and d are relatively prime with $bd+1$. Consider how we could have an e such that a is a power of b, c is a power of d, e is a power of $bd+1$, ae is a power of $b(bd+1)$ and ce is a power of $d(bd+1)$.