Math 116b - Homework 4

`Instructor:` Andrés Eduardo Caicedo

`Due:` February 12, 2008 at 1:00 pm.

This Homework is due during lecture by Tuesday February 12 at 1:00 pm. Refer to the grading policy for additional requirements.

1. Show that the function $Subs$ is recursive, where if $a = \ulcorner \varphi \urcorner$, $b = \ulcorner x \urcorner$ and $c = \ulcorner y \urcorner$ for some formula φ and variables x, y, then $Subs(a, b, c)$ is the Gödel number of the formula resulting from substituting each free occurrence of x by y in φ, and if a, b, c are not as described, then $Subs(a, b, c) = 0$.

   Use this to show that *diag*, as defined in lecture, is recursive.

2. Show that $f : \mathbb{N}^k \to \mathbb{N}$ is represented in Q iff it is Σ_1-represented in Q.

3. Show that $A \subseteq \mathbb{N}$ is recursive and infinite iff there is a total recursive $g : \mathbb{N} \to \mathbb{N}$ such that $A = \mathrm{ran}(g)$ and A is listed by g in increasing order.

4. Show that there are disjoint Σ_1 sets A and B for which there is no Δ_1 set C such that $A \subseteq C$ and $C \cap B = \emptyset$. One usually calls such a pair A, B of r.e. sets, *recursively inseparable*.