Math 116b - Homework 2

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`Due:` January 29, 2007 at 1:00 pm.

This Homework is due during lecture by Tuesday January 29 at 1:00 pm. Refer to the grading policy for additional requirements.

1. Show that every finite and every co-finite subset of $\mathbb{N}^k$ is r.e. Show that if $A, B \subseteq \mathbb{N}^k$ and A and B are r.e., so is $A \cap B$. The same is true for $A \cup B$, but this is harder. (In case you find one, feel free to provide a proof of this fact.)

2. Let $rem(x, y)$ be the remainder on dividing x by y, given by

$$rem(x,y) = \begin{cases} \text{the unique } z \text{ such that } \exists w \leq x\, (yw + z = x \text{ and } z < x), \text{ if } y \neq 0 \\ 0, \text{ otherwise.} \end{cases}$$

   We say that a (number coding a) pair a, m of natural numbers codes a sequence $x_0, x_1, \dots$ of natural numbers iff

$$x_i = rem(a, m(i+1)+1).$$

   The *Chinese remainder theorem* states that whenever $k \in \mathbb{N}$, $z_1, \dots, z_k \in \mathbb{N}$ are given, and $y_1, \dots, y_\kappa \in \mathbb{N}$ are pairwise coprime, then there is $a \in \mathbb{N}$ such that

$$a \equiv z_i \pmod{y_i}$$

   for all $i = 1, \dots, k$.

   Use the Chinese remainder theorem and the above to provide a different proof of Gödel's lemma from the one given in class, i.e., use the above to find a Δ_0 formula $\theta(a, b, c)$ stating that "the b^{th} number coded by a is c."

3. Find a model of Robinson's Q in which $<$ is not interpreted as a linear order.

4. By considering $\mathbb{Z}[X, Y, Z]/(XZ - Y^2)$ ordered by $0 < X < Y < Z$ obtain a model of Q where the statements "x is prime" and "x is irreducible" are not equivalent.